Image credit: Hubble telescope

Asymmetric Correlation Functions

Beyond LCDM

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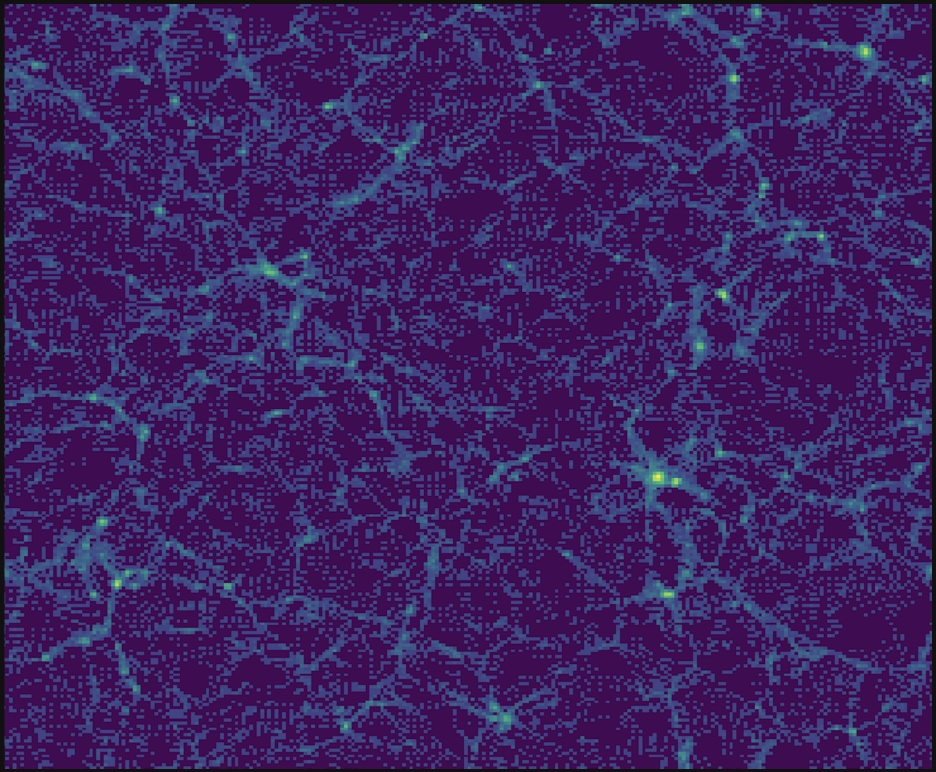
Structure growth (LCDM)

- Inflation seeds a stochastic field

$$P_{\mathcal{R}}(\boldsymbol{k}) = \frac{2\pi^2}{k^3} \mathcal{A}_s (k\eta_0)^{n_s - 1} \quad , \quad \left(\mathcal{R} = -\frac{aH}{\bar{\phi}'} \delta\phi \right)$$

- The perturbations then evolve deterministically according to currently known physical laws.
- These two assumptions yield predictions for the properties of the observed density fields.



Correlation Functions

- A stochastic distribution can be inferred from a sample through its moments

$$\langle X^n \rangle = \int P(X) X^n dX$$

- A stochastic field is defined on a space of points, where each point has a degree of freedom that is sampled from the distribution. The points may be non-trivially related; Their relationship can be inferred using autocorrelation functions of the field

$$\langle X(\vec{x}) X(\vec{x}+\vec{d}) \rangle \equiv \xi(\vec{d}) = \int \frac{d^n x}{V^{(n)}} X(\vec{x}) X(\vec{x}+\vec{d})$$

- In our case, physics provides several tracers of the stochastic field, whose relations contain information about the physics that acts on on them. We can quantify their statistical relationship using cross-correlation functions

$$\langle Y(\vec{x}) Z(\vec{x}+\vec{d}) \rangle \equiv \xi(\vec{d}) = \int \frac{d^n x}{V^{(n)}} Y(\vec{x}) Z(\vec{x}+\vec{d})$$

Correlation Functions

- A Gaussian random field (GRF) has each point sampled from a Gaussian distribution. The points may be correlated. We can picture all the points as an infinite-dimensional vector

$$P(\vec{X}) = \frac{\exp\left(-\frac{1}{2}(\vec{X}-\vec{m})^T \boldsymbol{C}^{-1}(\vec{x}-\vec{m})\right)}{(2\pi)^{N/2}(\det \boldsymbol{C})^{1/2}}$$

- In the event of a GRF, we can uniquely determine it from its two first moments, or equivalently, from the mean vector and correlation matrix.
- Problem of tiny sample-size at each point is answered by cosmological principle and assumption of ergodicity.

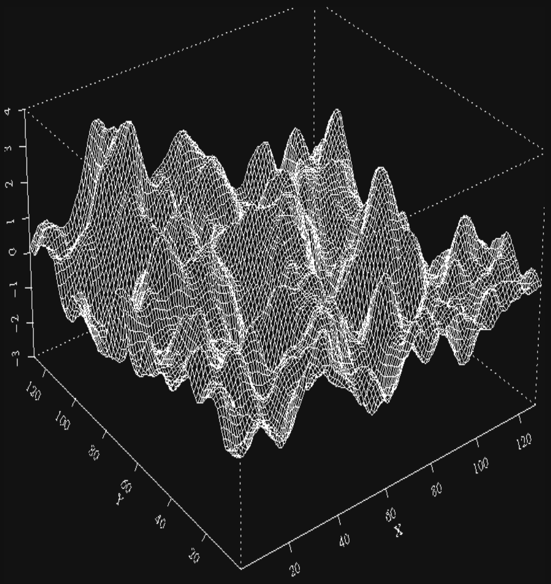


Image credit: Ben Kedem, Maryland university

Survey Distortions

- In LCDM perturbation theory, we can relate the dark matter density field to the primordial curvature perturbations whose distribution we know from inflation. In Fourier space

$$D(k,\eta) = \mathcal{F}(\delta(x,\eta)) = -\frac{2}{3\Omega_m}(\frac{k}{\mathcal{H}_{\prime}})^2 D(a)T(k)\sqrt{P_{\mathcal{R}}}$$

- Furthermore, using for example the Press-Schechter formalism, we can relate the number densities of non-linear structure, like halos or galaxies, to that of the dark matter density field. At first order $\delta_g = b\delta_{cdm}$
- Galaxy/halo counts can be done in satellite/telescope surveys. However there are complications in making the correct counts owing to survey volume distortions, due to among other propagation effects [Yoo et al. 2012, Bonvin and Durrer 2012].

Survey Distortions

- The Doppler effect changes the distance one would infer for the galaxy when one uses the redshift-distance relation.

  Normally $r^{(c)} = \int \frac{cda}{a^2 H}$ and $a = \frac{1}{1+z}$, and we measure z from spectrography.

  Now, $a = \frac{\lambda_{emit}}{\lambda_{observ}} = \frac{\lambda_0(1+\vec{v}_s \cdot \hat{n})}{\lambda_0(1+z)(1+\vec{v}_O \cdot \hat{n})} \simeq \frac{1}{1+z}\,(1+(\vec{v}_S - \vec{v}_O)\cdot \hat{n})$

- This translates to a distortion in the overdensity $\Delta^{std} = b\delta - \frac{1}{\mathcal{H}}\partial_r(\vec{v}\cdot\hat{n})$
- If we go to Fourier space, we recover the famous Kaiser formula [Kaiser 1987] (note angular dependency)

$$P_{XY}(k) = \langle \Delta_X \Delta_Y \rangle = P_{\delta\delta}(k)(b_X + f\mu^2)(b_Y + f\mu^2)$$

Survey Distortions

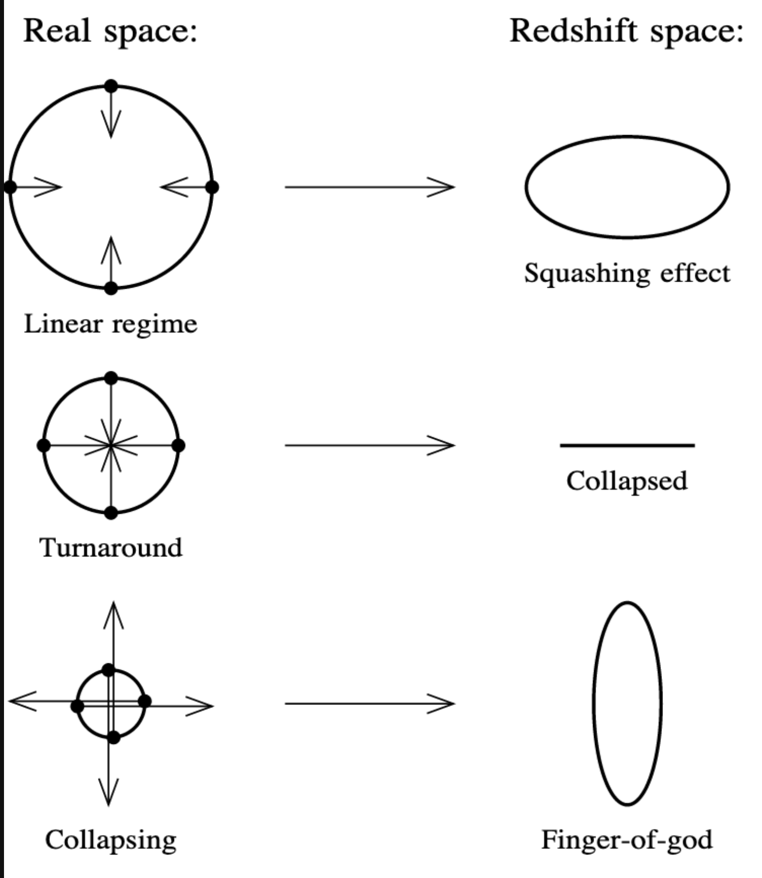


Image credit: Hamilton (1997)

Legendre Projection

- Natural function basis to separate different physical effects, since different terms derived by similar method to the above comes with different angular dependencies.
- Application together with cleverly chosen tracers for cross-correlation enhances our ability to resolve degeneracies of different effects.
- In the small-angle approximation $\cos\phi \simeq \mu$

$$\mathrm{Proj}_{\mathrm{Legendre}}(f(x,\mu)) = \frac{(2l+1)}{2}\int_{-1}^{1} d\mu f(x,\mu)\mathcal{P}_l(\mu)$$

- Odd multipole contributions? [Bonvin et al. 2014]

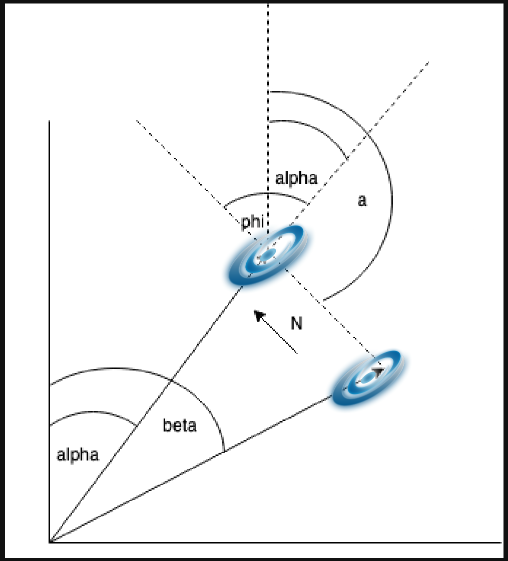
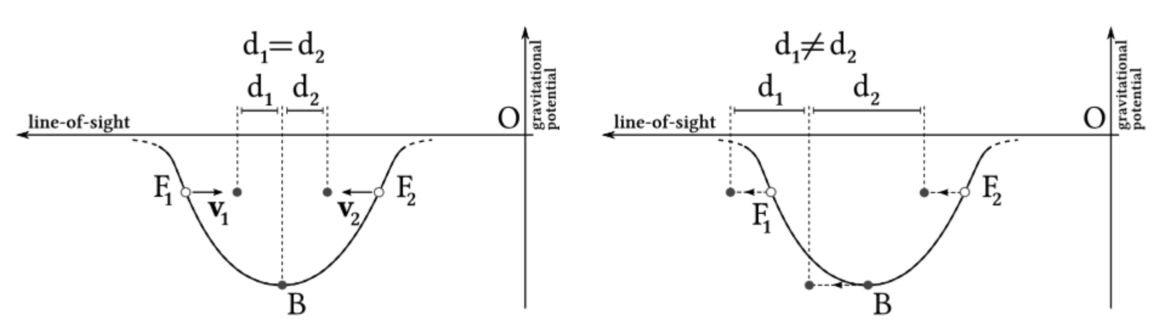


Image credit: Bonvin et al 2014

$$\Delta^{(rel)} \supset \mathcal{H}^{-1}\partial_r\psi$$



$$\Delta^{(rel)} = \mathcal{H}^{-1}(\partial_r\psi + \dot{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}) - \left[\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}} - 1 + 5s\left(1 - \frac{1}{r\mathcal{H}}\right)\right]\boldsymbol{v}\cdot\hat{\boldsymbol{n}}$$

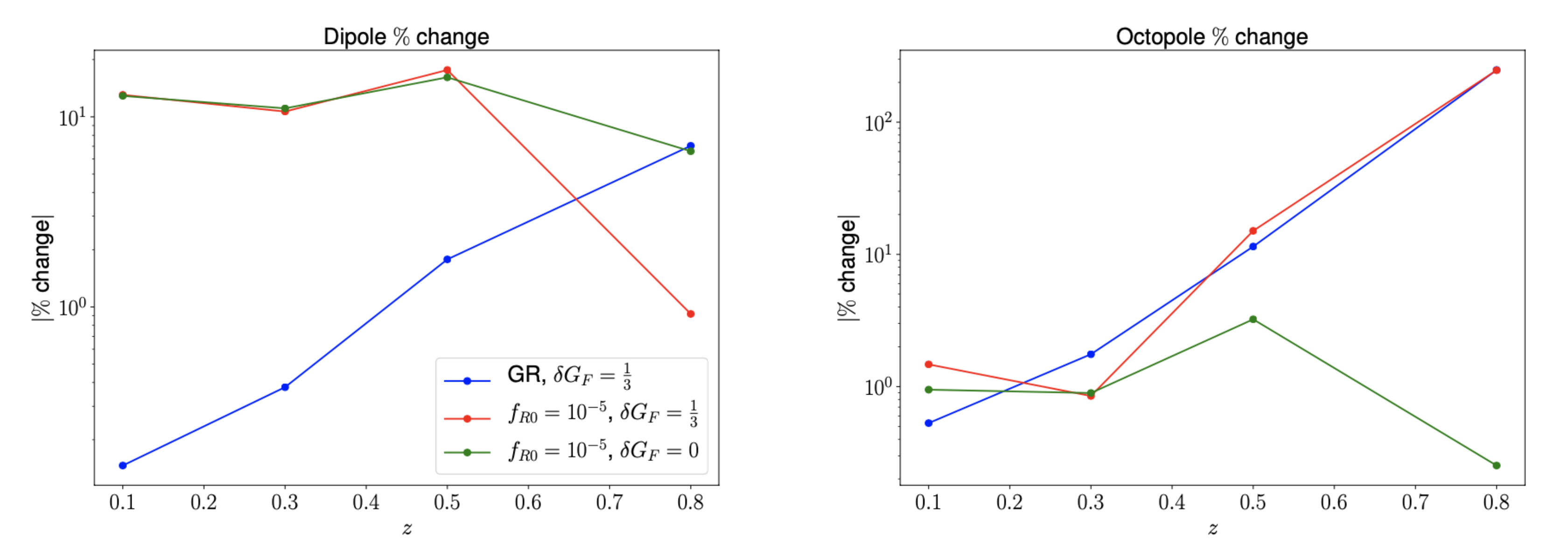
$$\Delta^{\text{lens}} = (5s-2)\int_0^r dr' r' \left(\frac{r-r'}{2r}\right)$$

$$\Delta^{\text{AP}} = (\partial_r - \partial_\eta)\left[\sum_{i\neq \text{AP}} \Delta^{(i)}\right]\frac{dr}{d\vec{\Theta}}\delta\vec{\Theta}$$

$$\Delta = \sum \Delta^{(i)}$$

$$\partial_r \Psi \to (1 + \delta G)\, \partial_r \Psi,$$

$$\Delta^{\mathcal{F}}(z, \hat{\mathbf{n}}) = \zeta \left[\frac{\dot{\mathbf{v}} \cdot \hat{\mathbf{n}}}{\mathcal{H}} + \mathbf{v} \cdot \hat{\mathbf{n}} \right]$$



Kodwani & Desmond (2019)

Effects of Beyond LCDM

- Since the odd multipoles seem sensitive to relativistic corrections, we might ask whether they would be to gravity modifications as well.

  → Equivalence principle violations/screening

  [Bonvin and Fleury 2018, Kodwani and Desmond 2019]

  → Enhanced/suppressed clustering

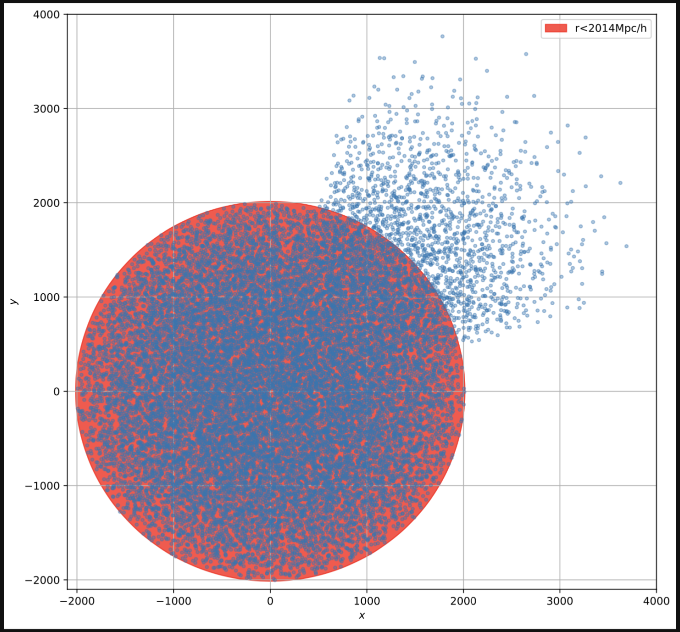
  → Propagation effects

  → Other survey volume distortions

- Unique and clean signature? What are optimal tracers to cross-correlate?

Search for signatures in simulations

- Previous studies [Breton et al. 2019, Beutler and Dio 2020, Guandalin et al. 2021]
- Gevolution is a good choice for modelling of relativistic effects.
- Extracting observables on the lightcone is conceptually simplest as we are interested in distortions on the lightcone.
- We consider halos at first.

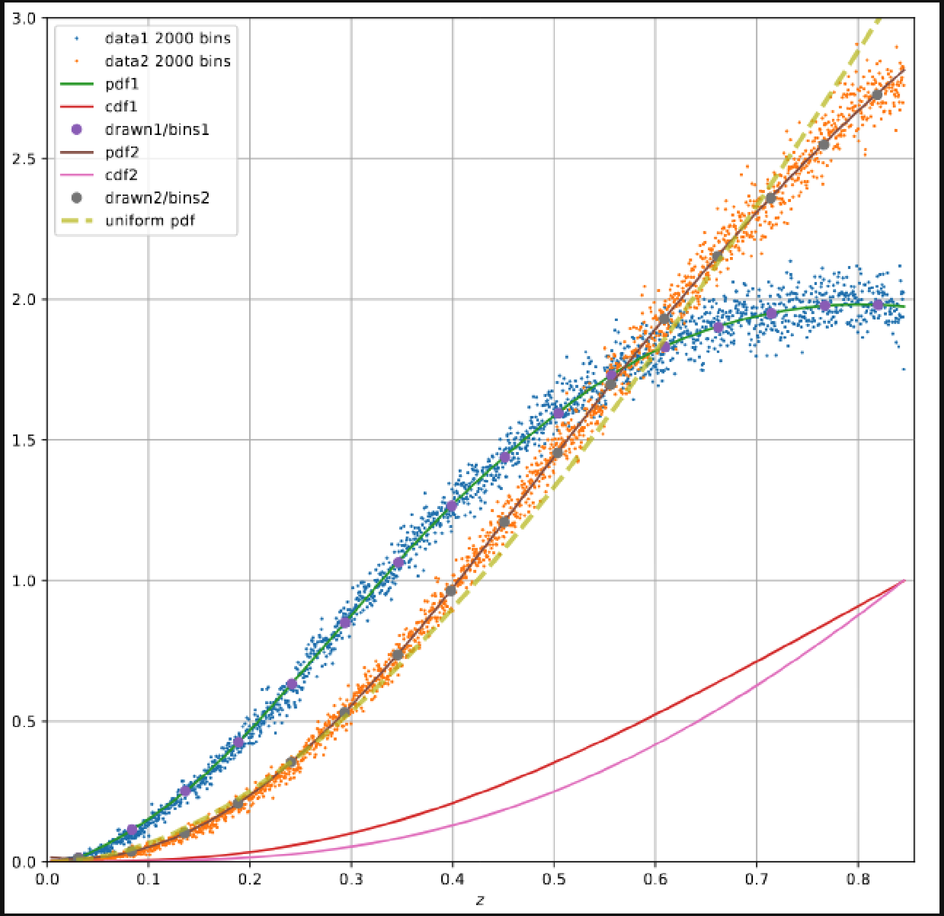


Constructing a random catalog

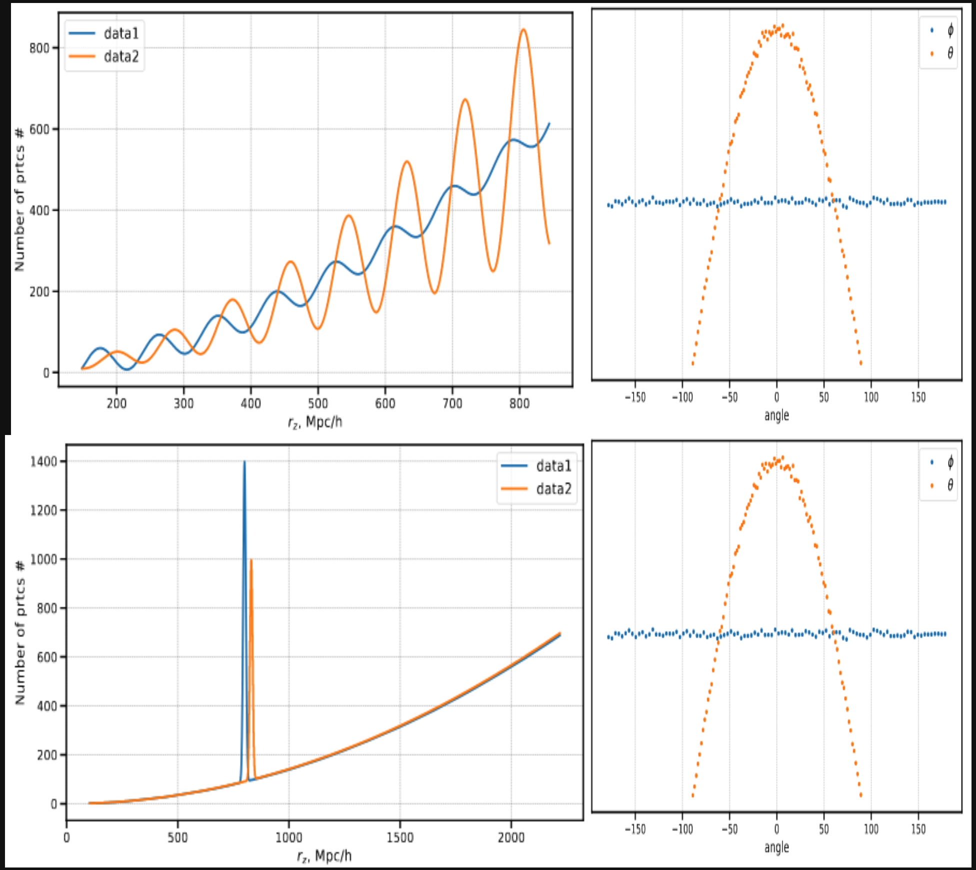
- Landay-Szalay estimator

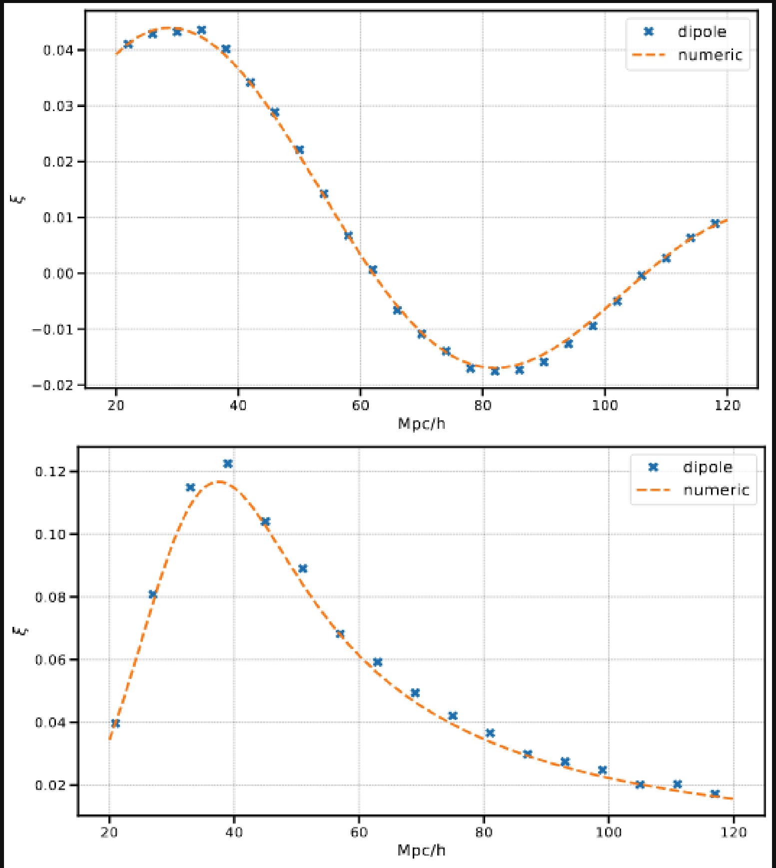
$$\xi_{LS} = \frac{\langle (D_1 - R_1)(D_2 - R_2) \rangle}{\langle R_1 R_2 \rangle}$$

- Uniform distribution?
- Account for halo number evolution?



Artificial Datasets





Summary

- Inflation provides us with stochastic field.
- We can extract information through moments of tracers.
- Tracers are imprinted by physics.
- Estimators are distorted by physics.

→ Hopefully we can knead the data in a way to glean new physics and resolve all degeneracies.

- Towards this aim we may

→ Choose different cross-correlations
→ Consider different statistical moments
→ Vary how we model our random sets
→ Project onto different functional bases
→ Subdivide and filter our data as we see fit
→ Weighted estimators

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