Imprints of Primordial Magnetic Fields in Dark Matter

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Observing primordial magnetic fields through Dark Matter

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Dark Matter
(DM)
Primordial Magnetic Fields
(PMF)

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- This mode may be observed, even if $B < 10^{-11}$ G !

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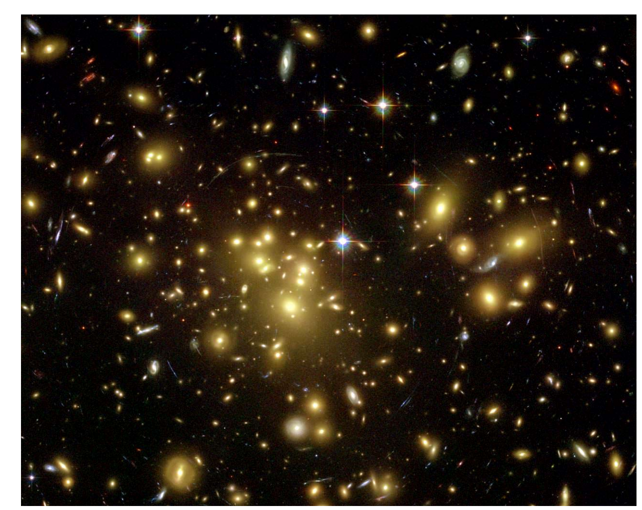

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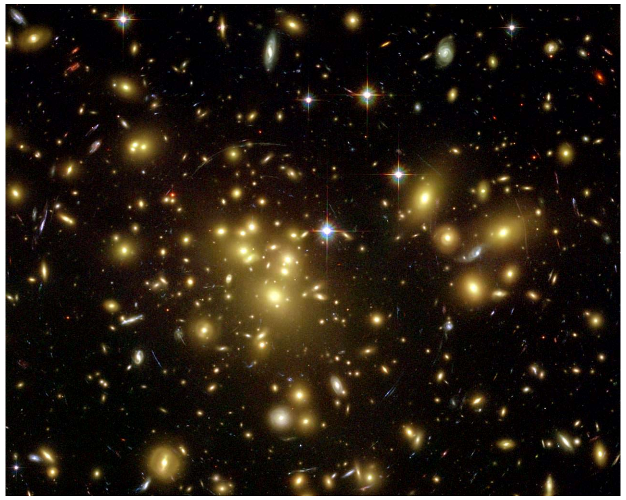

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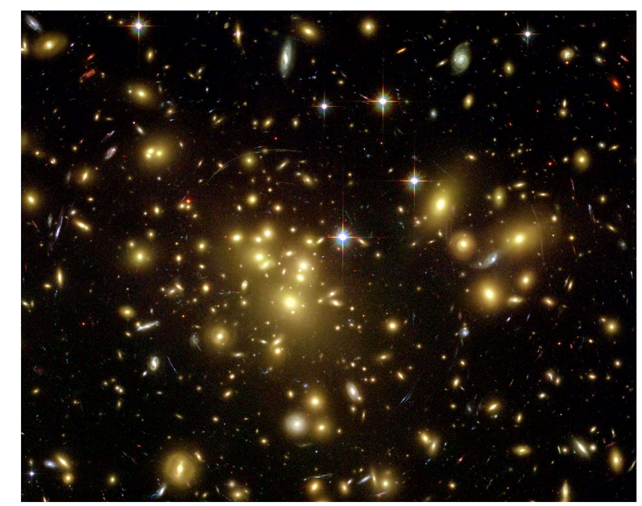

$$B_{brain} \sim 10^{-10}\,\mathrm{G}$$

Magnetic Field in Curved Universe

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Energy density of the magnetic field

$$\rho = -\frac{1}{2} B^\mu B_\mu = \frac{1}{2} \mathbf{B}^2$$

Stochastic Magnetic Field

$$\left\langle B_i(\mathbf{k},t)\, B_j\left(\mathbf{q},t\right)\right\rangle = (2\pi)^3\, \delta\left(\mathbf{k}+\mathbf{q}\right) \left[\left(\delta_{ij} - \hat{k}_i\hat{k}_j\right) P_B\left(k,t\right) - i\epsilon_{ijm}\hat{k}_m P_{aB}\left(k,t\right)\right]$$

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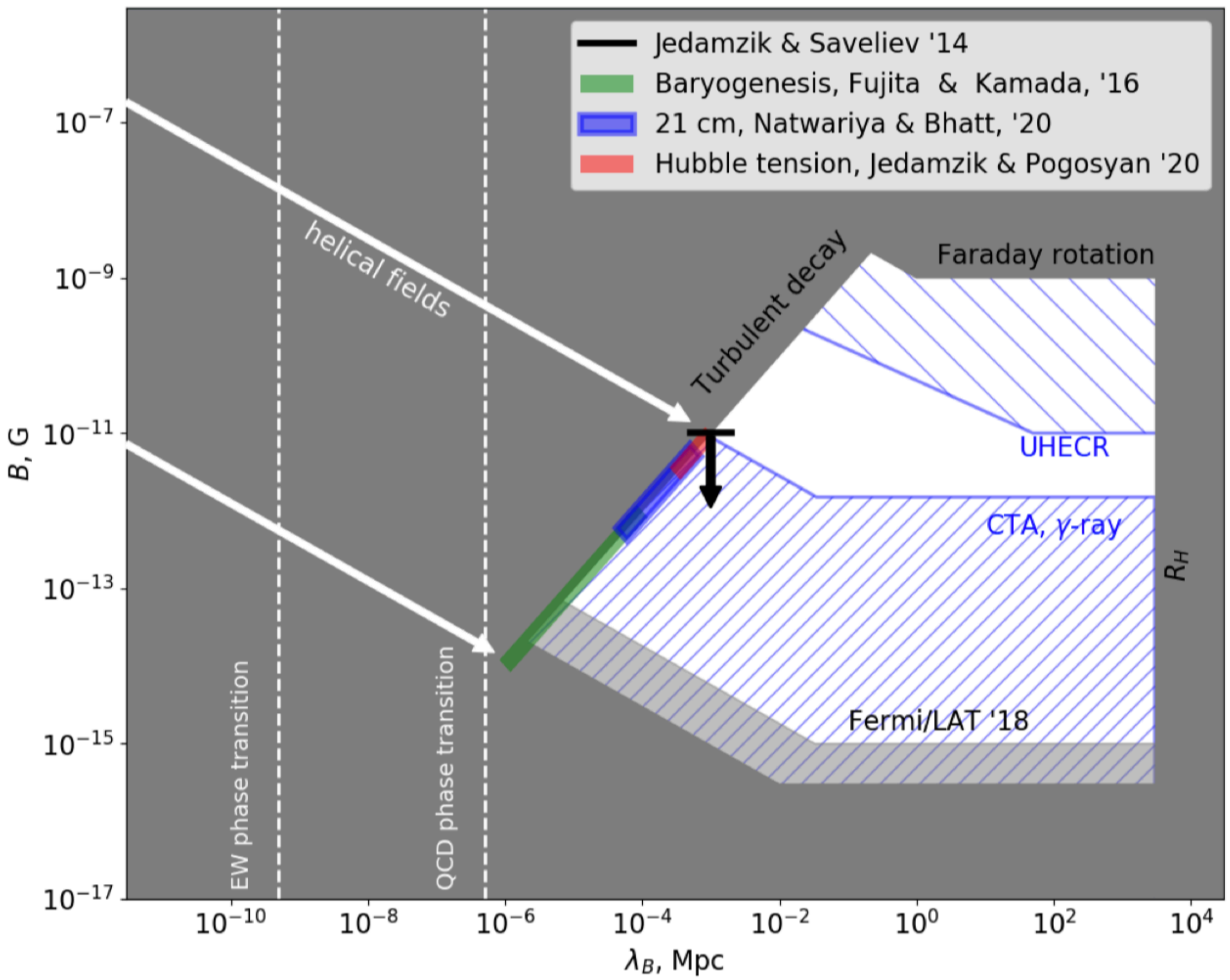


Korochkin et. al. (2020)

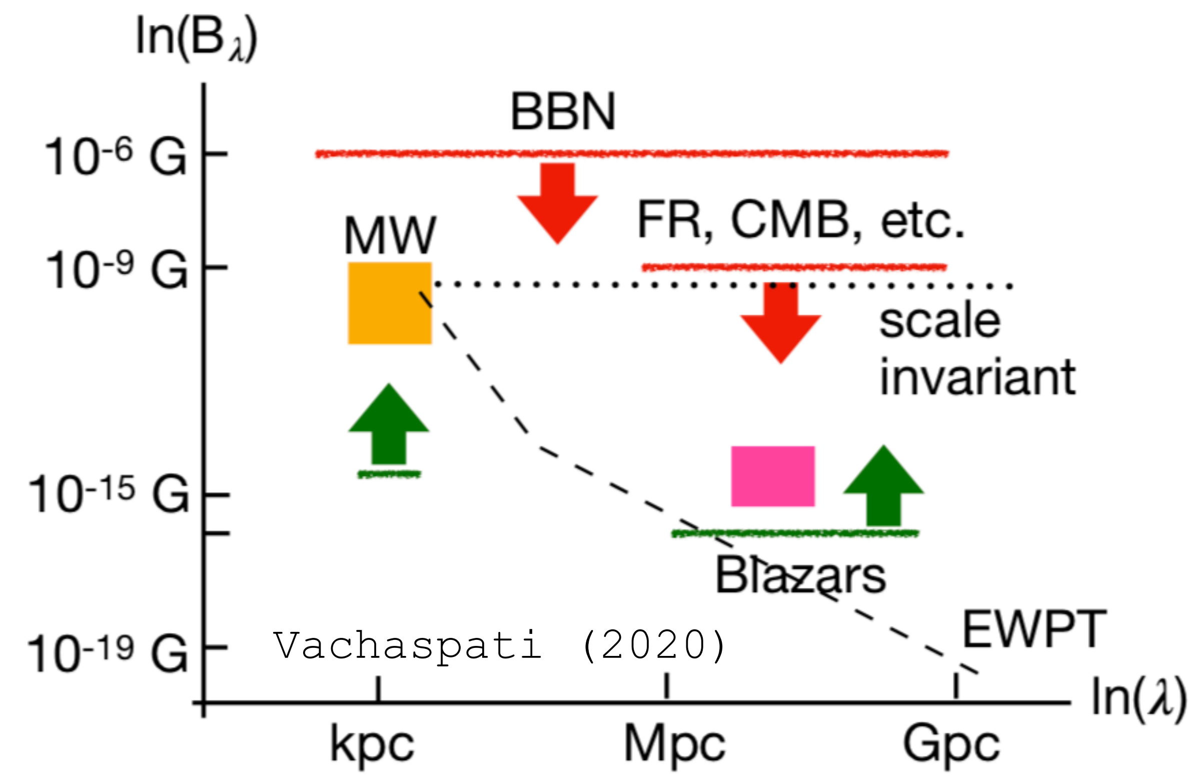
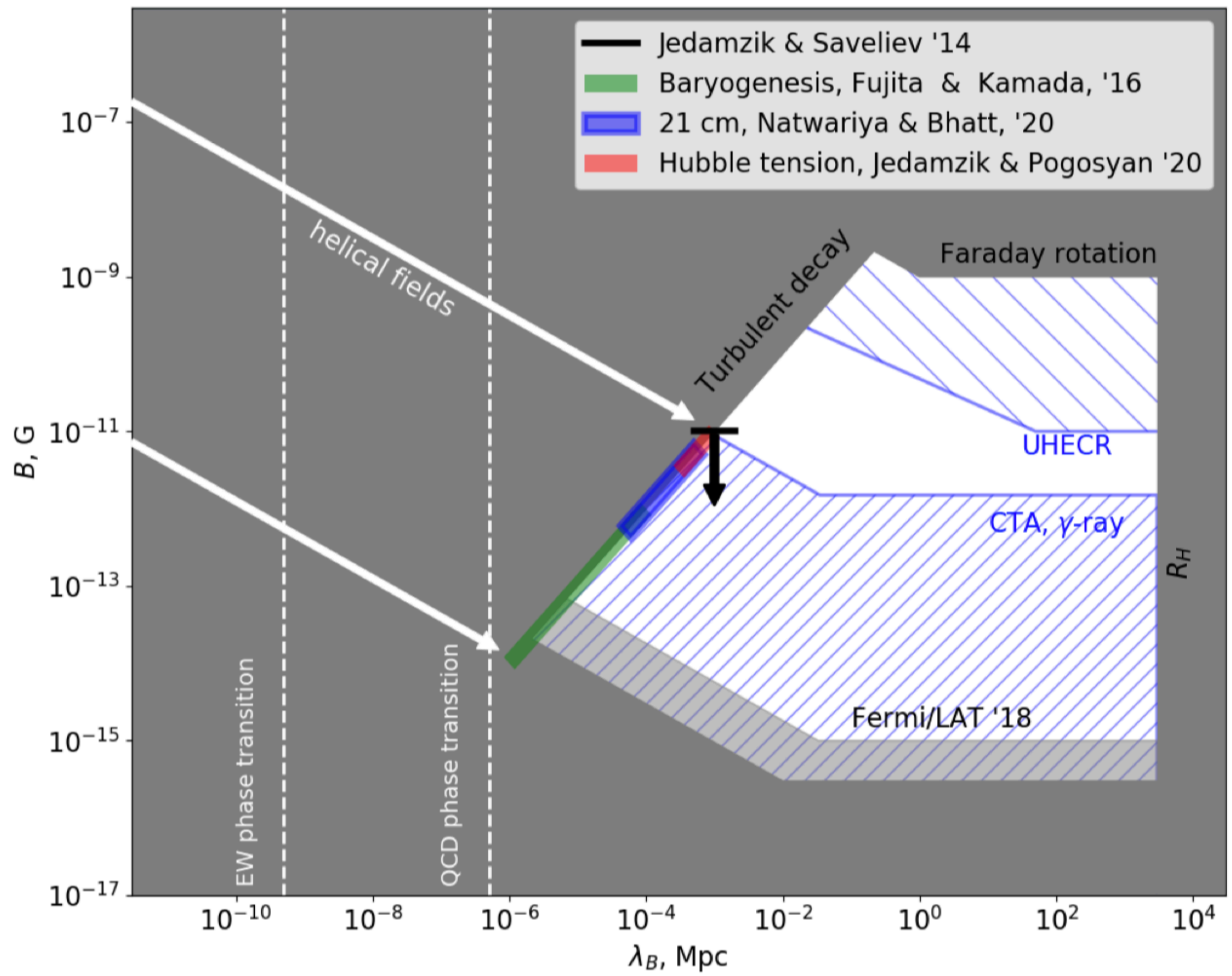


FIG. 17: Cosmological magnetic fields with $\lambda \sim$ kpc and $B_\lambda \sim 10^{-10}$ G may directly explain, *i.e.* with minimal dynamo amplification, the galactic magnetic field, and may help resolve the present Hubble tension due to baryon clustering at Hydrogen recombination. This region is denoted by the golden rectangle marked by "MW" (for Milky Way). Big bang nucleosynthesis (BBN) constrains $B_\lambda \lesssim 10^{-6}$ G on all scales, while other observations roughly constrain $B_\lambda \lesssim 10^{-9}$ G on Mpc to Gpc scales. Blazar spectral measurements place a lower bound $\sim 10^{-16}$ G for λ in the Mpc to Gpc range, or instead a lower bound of $\sim 10^{-15}$ G on kpc scales, and assumes that plasma instabilities do not play a role. Magnetic helicity measurements are uncertain but, if confirmed, would lie in the region of the pink rectangle. If the cosmological magnetic field is scale invariant, as in some inflationary models, and also passes through the golden rectangle, it would correspond to the horizontal dotted line and is pressured by several constraints. If the cosmological magnetic field has a blue k^3 or k^4 spectrum as might be expected from the EWPT, it may correspond to a shape like the dashed curve. This constraint plot is meant to be schematic as there is considerable uncertainty in the numbers and also the caveats mentioned in the text. For example, B_λ may be below the blazar bound shown by the green line at $\lambda \sim$ Mpc since small scale (kpc) magnetic fields can instead explain the absence of blazar halos.

Power spectrum $P(k)k^2$



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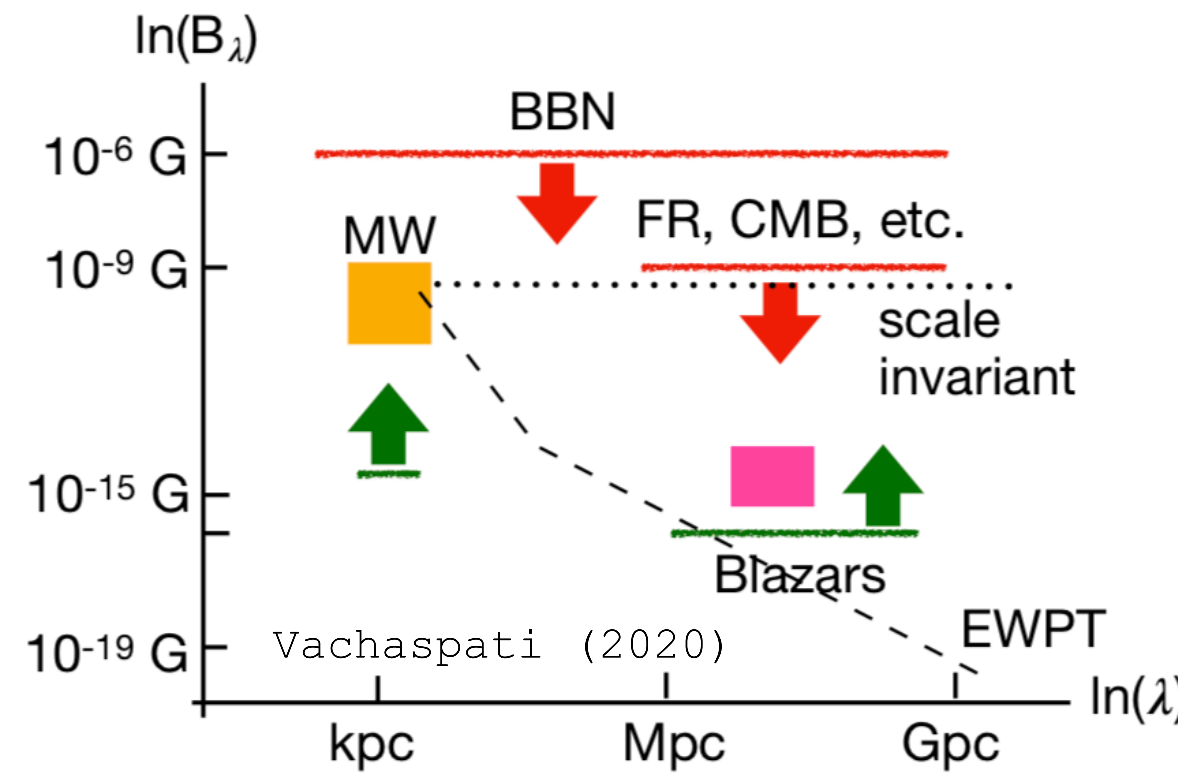

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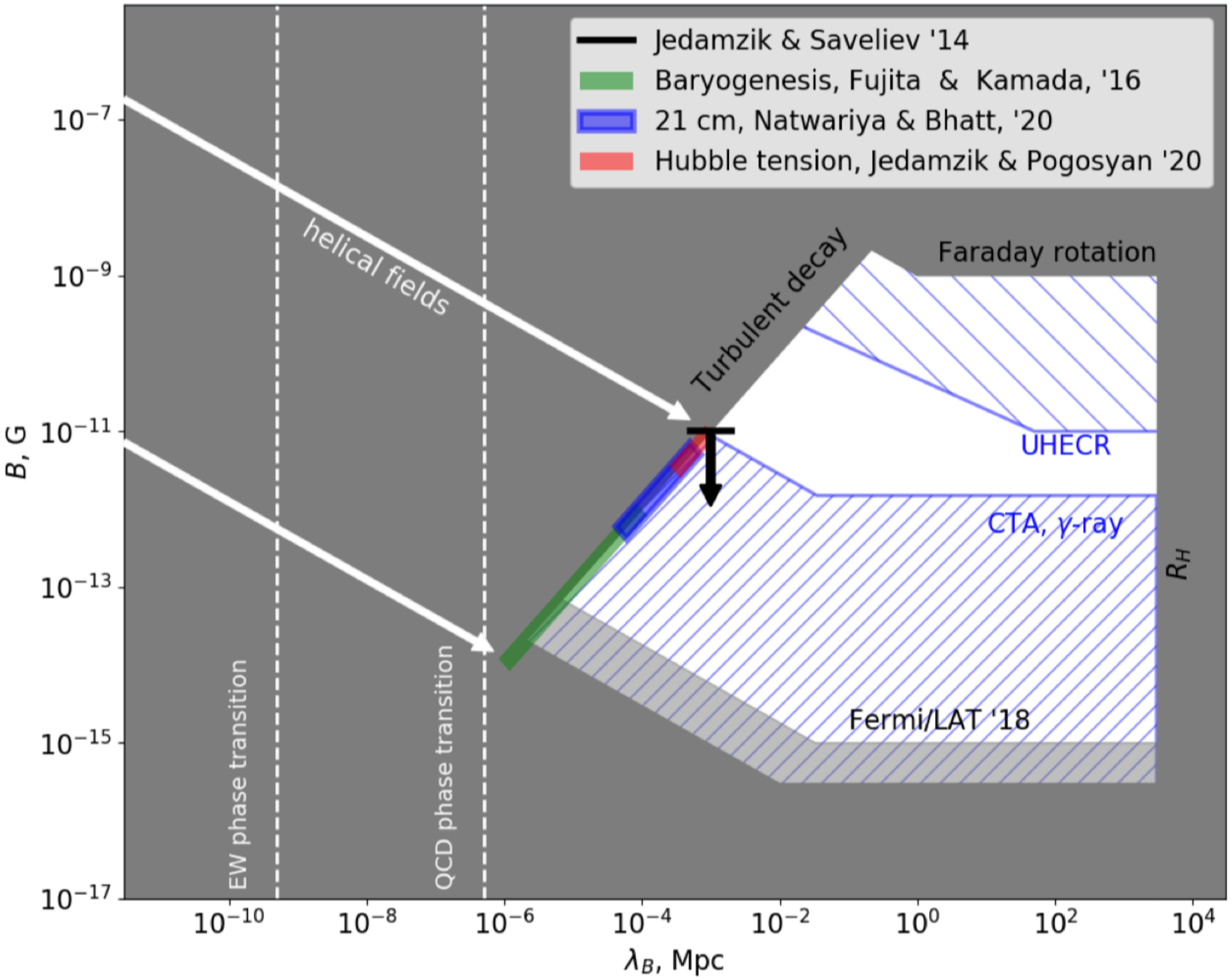

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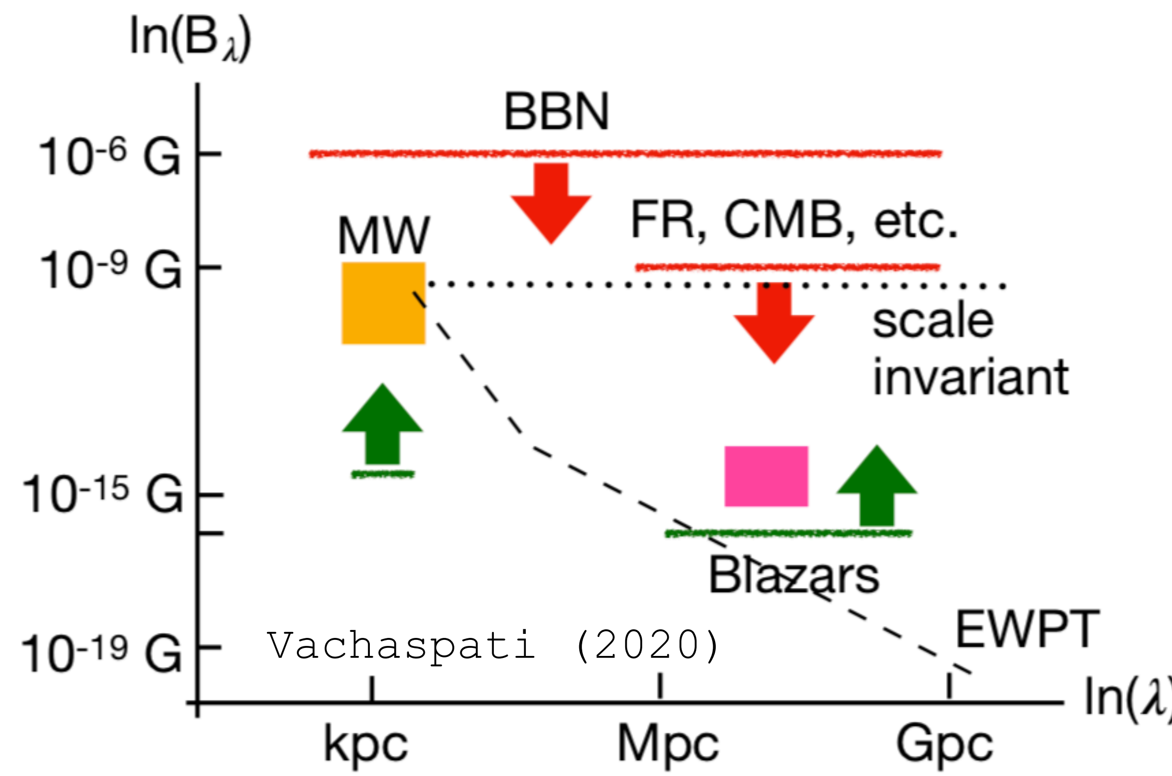

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$$\mathscr{L} = \frac{(\partial\chi)^2}{2} - \frac{\left(M^2 - \mu^2(t,\mathbf{x})\right)\cdot\chi^2}{2} - \frac{\lambda\chi^4}{4}$$

Babichev, Gorbunov, Ramazanov(2020)

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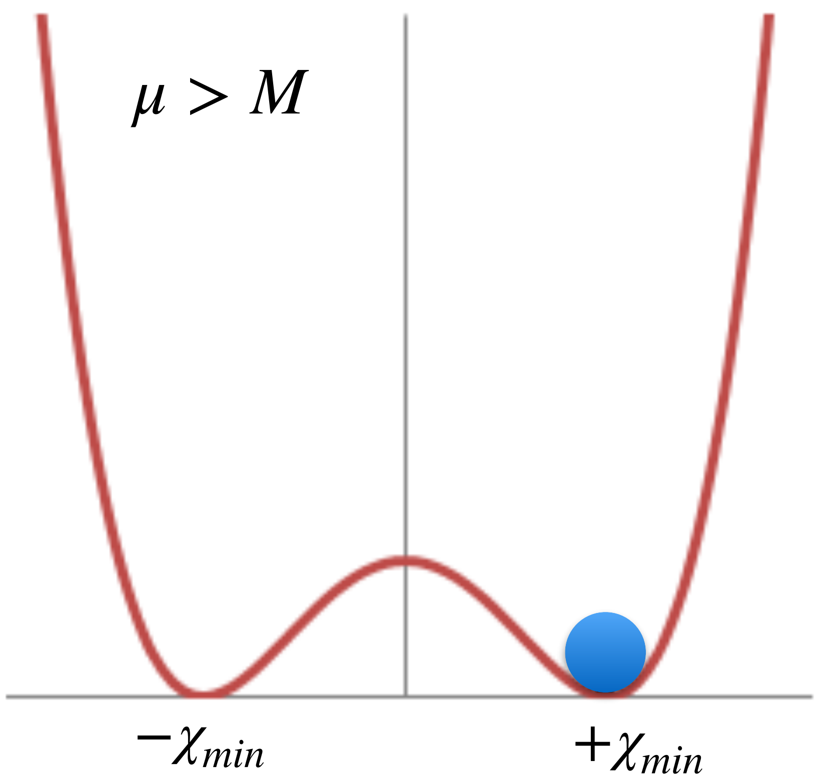
Early universe spontaneously Brocken Phase



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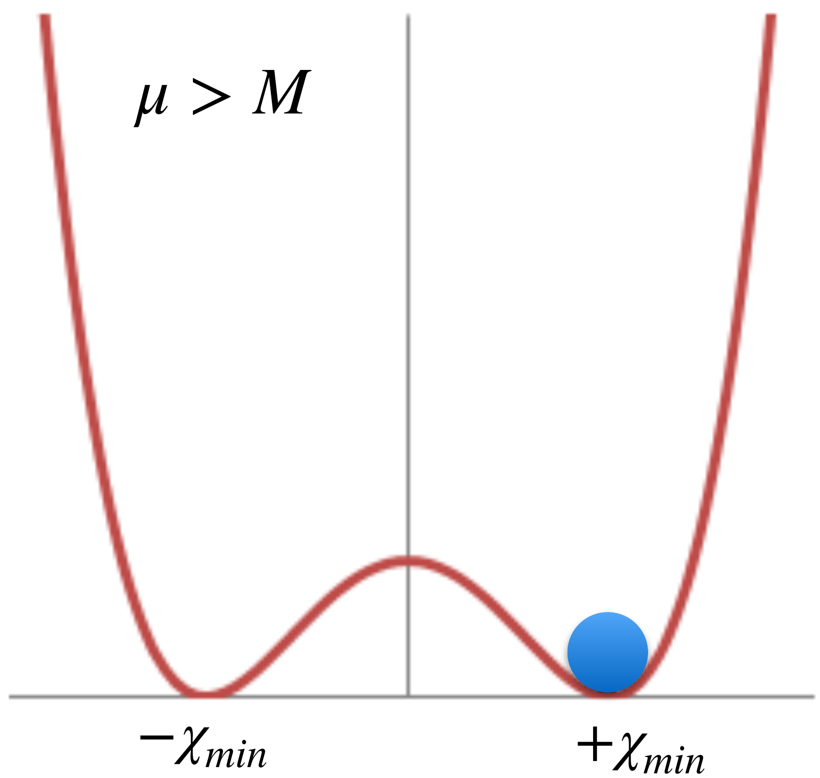

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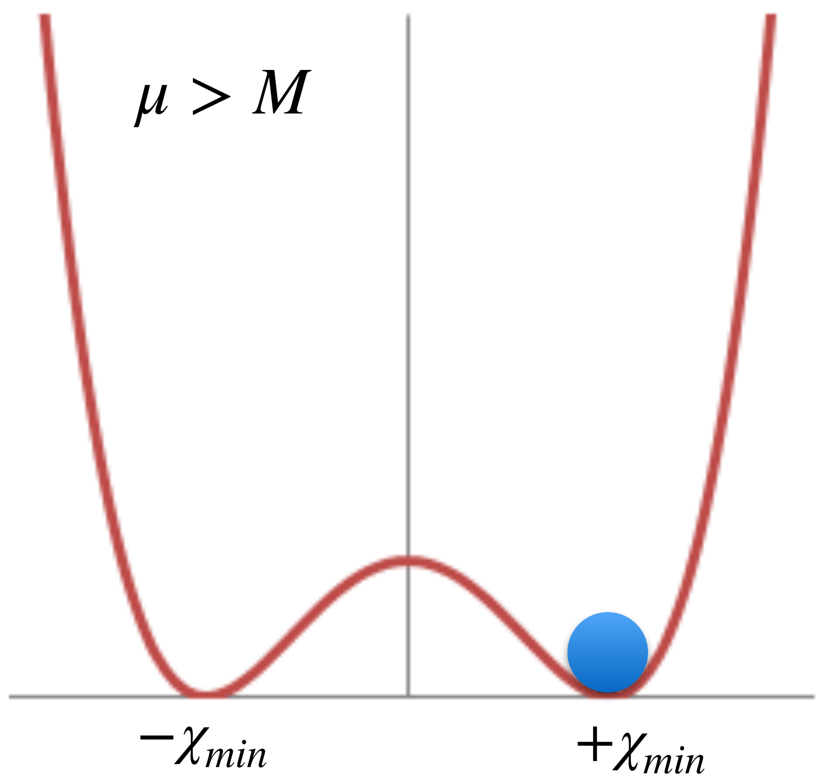

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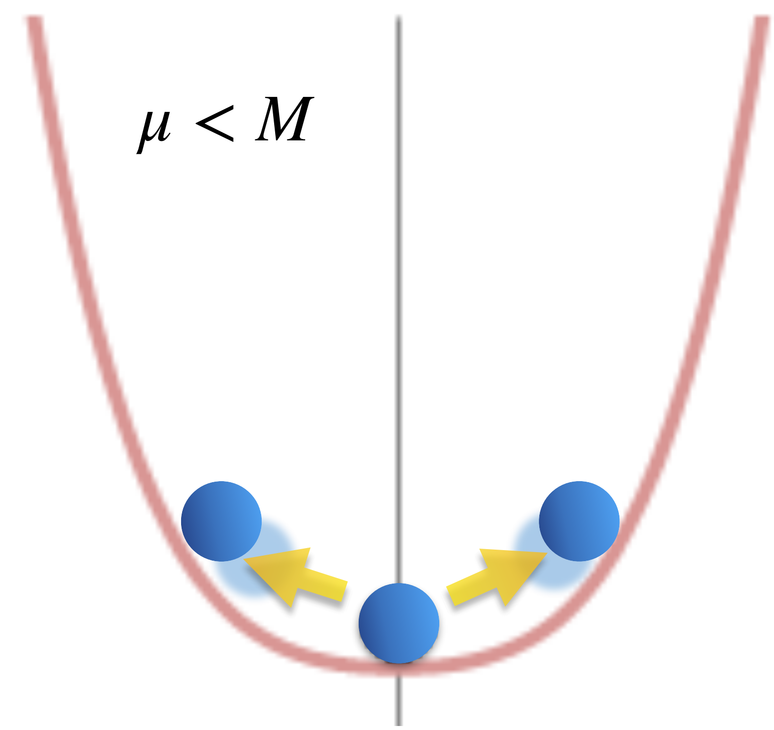
Late universe oscillations around restored symmetric vacuum



for Hubble parameter

$$H < M$$

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At this point one cannot trace the minimum as $\dot{\chi}_{min} = \frac{\mu\dot{\mu}}{\sqrt{\lambda\left(\mu^2(t) - M^2\right)}}$ diverges**!**

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adiabaticity is violated at $\mu \simeq M$, as $M_{eff} \simeq 0$, then if $M > H_*$ the field starts to oscillate with amplitude

$$\chi_* \simeq \frac{\left(2M^2\right)^{1/3}}{\sqrt{2\lambda}} \left|\frac{\dot{\mu}}{\mu}\right|_*^{1/3} \simeq \frac{(\kappa H_* M^2)^{1/3}}{\sqrt{2\lambda}}$$

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for the model of the paper $\kappa = 4$

Repetitio est mater studiorum

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John William Strutt

cf.

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How dark is DM?

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Background Magnetic Field

$$\mathbf{B}^2(t) = \int_{\frac{k}{a_0}<H_0} \frac{k^2\, dk\, P_B(k)}{\pi^2} \propto \ln \frac{H_0}{\Lambda_{IR}}$$

$$\mathbf{B} = \mathbf{B}\,(t) + \delta\mathbf{B}\,(t, \mathbf{x})$$

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Planck collaboration(2018)

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* only for normal isocurvature without statistical anisotropy

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For $\delta B/B \simeq 1$ the fraction is $f \lesssim 10^{-6}$

When does it work?

e.g. $M = 0.5$ TeV

$\Lambda = 0.4 \cdot 10^{14}$ GeV

$\lambda = 10^{-12}$

$T_{reh} = 10^{10}$ GeV

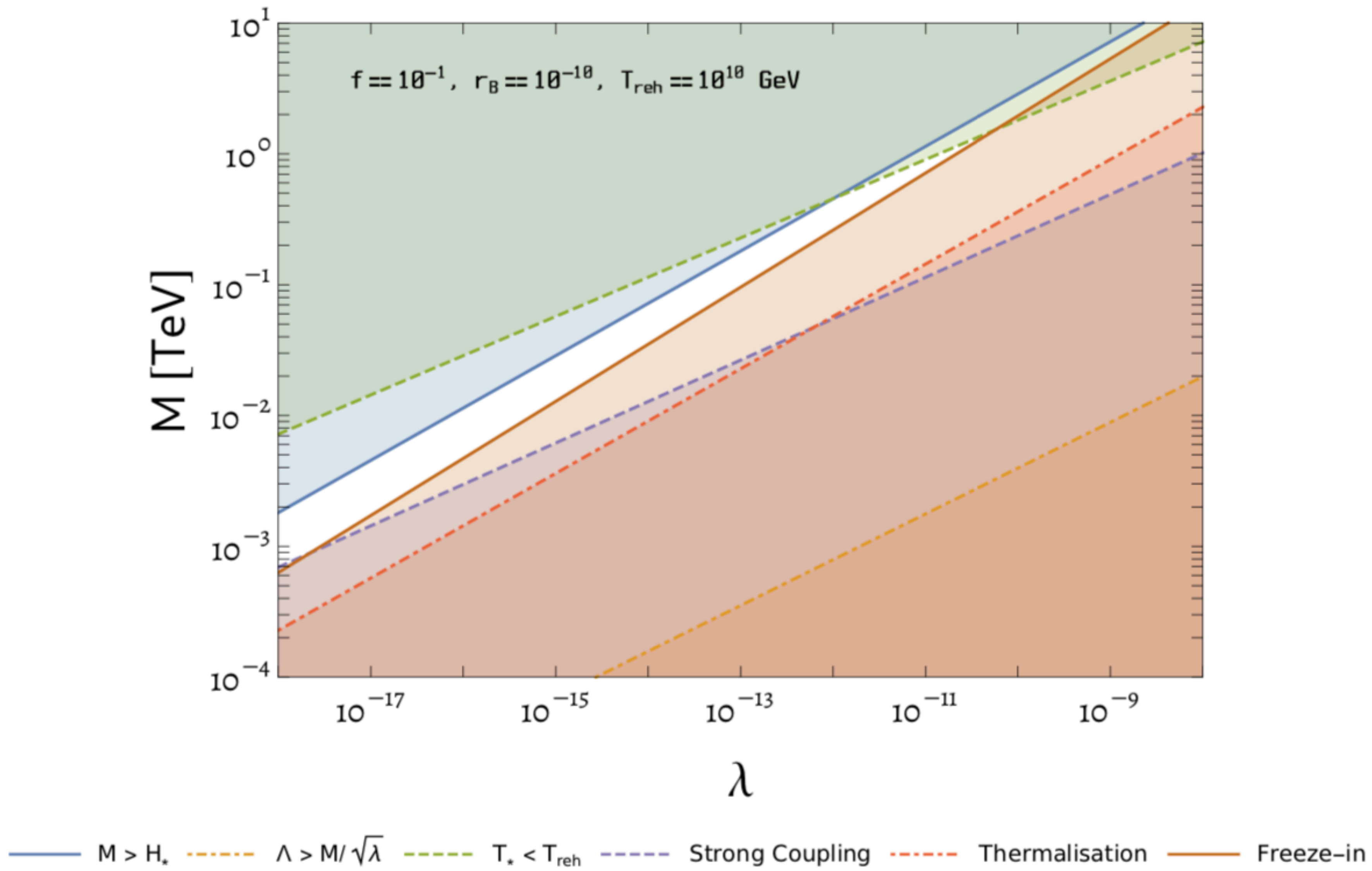

Figure 1: The exclusion plot shows allowed values (white region) for the mass M of the field χ and the self-interaction coupling constant λ. The other parameters, the reheating temperature T_{reh}, the DM fraction f defined by Eq. (35), and the present day magnetic to radiation energy density ratio $r_B(t_0)$ defined by Eq. (31), are set to the values shown in the plot. The number of ultra-relativistic degrees of freedom is fixed to $g_{*,reh} = g_*(T_*) = 10^3$. For values of the mass M and the coupling constant λ along the solid orange line, the rest of DM is made of particles χ produced through the freeze-in mechanism.

$$r_B = \frac{\rho_B}{\rho_r}$$

$r_B = 10^{-11}$ corresponds $B = 10^{-11}$ G

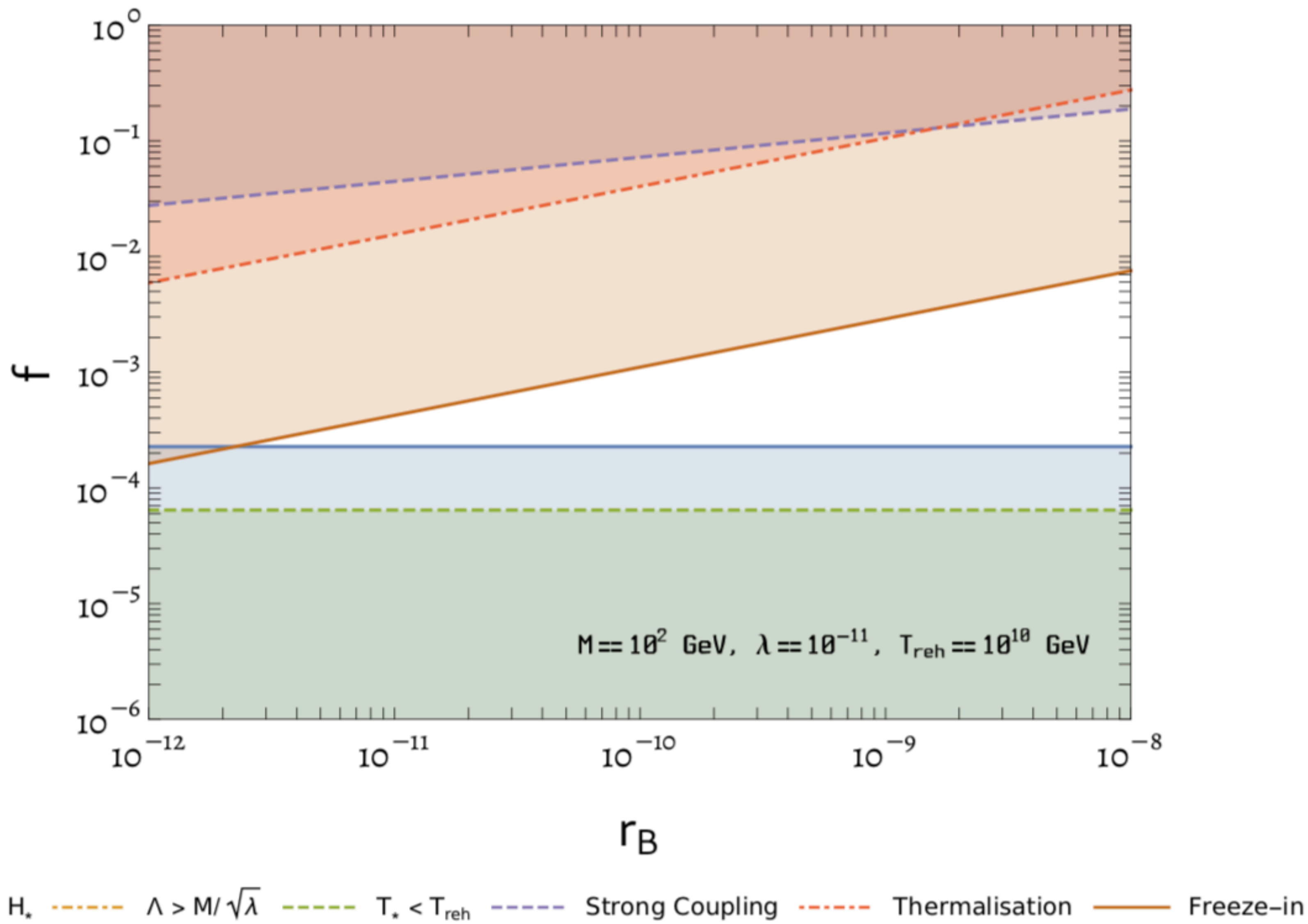

Figure 2: The exclusion plot shows allowed values (white region) of the DM fraction f defined by Eq. (35) and the present day magnetic to radiation energy density ratio $r_B(t_0)$ defined by Eq. (31). The other parameters, the reheating temperature T_{reh}, the mass M of the field χ, and the self-interaction coupling constant λ, are set to the values shown in the plot. The number of ultra-relativistic degrees of freedom is fixed to $g_{*,reh} = g_*(T_*) = 10^3$. For f and $r_B(t_0)$ taking values along the solid orange line, the rest of DM is made of particles χ produced through the freeze-in mechanism.